Princess Sumaya University for Technology
King Abdullah II School of Engineering

EE27355
Communication Principles

Quiz #5
Wednesday 1/4/2026

Name:………………………………………………………………. **Section 2**

Q.1) Use the frequency shift property and Table 3.1 to find the Fourier transform of the signal shown in Figure Q.1.

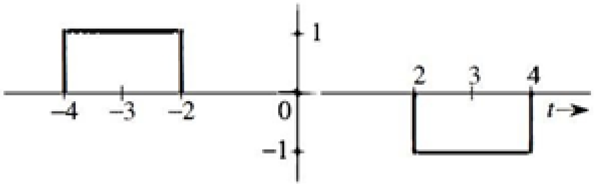

Figure Q.1

Solution: **[5-Points]**

$$g(t) = rect\left(\frac{t}{2}\right) \longleftrightarrow 2\ sinc\ (\omega)$$

$$rect\left(\frac{t}{\tau}\right) \longleftrightarrow \tau\ sinc\ \left(\frac{\omega\tau}{2}\right)$$

$$g(t+3) - g(t-3) \longleftrightarrow 2j[2\ sinc\ (\omega)\sin 3\omega] = 4j\ sinc\ (\omega)\sin 3\omega$$

Q.2) Use the frequency shift property and Table 3.1 to find the inverse Fourier transform of the spectra shown in Figure Q.2.

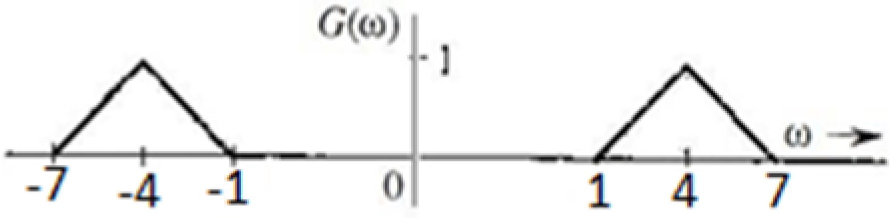

Figure Q.2

<u>Solution:</u> [5-Points]

$$G(\omega) = \Delta\left(\frac{\omega + 4}{6}\right) + \Delta\left(\frac{\omega - 4}{6}\right)$$

$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\,\text{sinc}^2\left(\frac{\omega\tau}{4}\right)$
$\frac{W}{2\pi}\,\text{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$

$$\frac{3}{2\pi}\text{sinc}^2\left(\frac{3}{2}t\right) \Longleftrightarrow \Delta\left(\frac{\omega}{6}\right)$$

W=3

$$g(t) = \frac{3}{\pi}\text{sinc}^2\left(\frac{3}{2}t\right)\cos 4t$$

Q.3) Use the frequency shift property and Table 3.1 to find the inverse Fourier transform of the spectra shown in Figure Q.3.

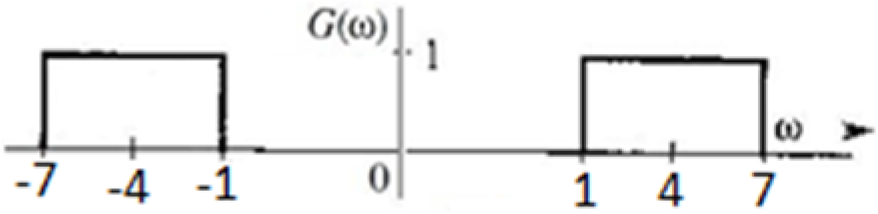

Figure Q.3

<u>Solution:</u> [5-Points]

$\mathrm{rect}\left(\frac{t}{\tau}\right)$	$\tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$
$\frac{W}{\pi}\,\mathrm{sinc}\,(Wt)$	$\mathrm{rect}\left(\frac{\omega}{2W}\right)$

W=3

$$G(\omega) = \mathrm{rect}\left(\frac{\omega - 4}{6}\right) + \mathrm{rect}\left(\frac{\omega + 4}{6}\right)$$

$$\frac{3}{\pi}\mathrm{sinc}(3\,t) \Longleftrightarrow \mathrm{rect}\left(\frac{\omega}{6}\right)$$

$$g(t) = \frac{6}{\pi}\mathrm{sinc}(\,3\,t)\cos 4t$$

Q.4) For the signal g(t)=[2a/(t^2+a^2)], determine the essential bandwidth B Hz of g(t) such that the energy contained in the spectral components of g(t) of frequencies below B Hz is 99% of the signal energy E_g.

<u>Solution:</u> [5-Points]

$$e^{-a|t|} \xrightarrow{F} \frac{2a}{a^2+\omega^2}$$

$$\frac{2a}{a^2+t^2} \xrightarrow{F} 2\pi e^{-a|\omega|}$$

$$E_g = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(\omega)|^2 d\omega$$

$$= \frac{1}{2\pi}(2)\int_0^{\infty}(2\pi)^2 e^{-2a\omega}\, d\omega$$

$$= \frac{4\pi^2}{\pi}\,\frac{-1}{2a}\, e^{-2a\omega}\Big|_0^{\infty}$$

$$= \frac{-4\pi^2}{2\pi a}\left[e^{-2a\infty} - e^{0}\right]$$

$$= -\frac{2\pi}{a}\left[0 - 1\right]$$

$$= \frac{2\pi}{a}$$

$$0.99\,\frac{2\pi}{a} = \frac{1}{2\pi}\int_{-W}^{W} |G(\omega)|^2 d\omega$$

$$= \frac{2}{2\pi}4\pi^2\int_0^{W} e^{-2a\omega}\, d\omega$$

$$= 4\pi\,\frac{-1}{2a}\, e^{-2a\omega}\Big|_0^{W}$$

$$0.99\,\frac{2\pi}{a} = -\frac{2\pi}{a}\left[e^{-2aW} - 1\right]$$

$$\frac{2\pi}{a}e^{-2aW} = \left[1 - 0.99\right]\frac{2\pi}{a}$$

$$e^{-2aW} = 0.01$$

$$\boxed{W = \frac{2.3025}{a}}$$

Answer / 2(3.14) = 0.366/a

Table 3.1

Short Table of Fourier Transforms

	$g(t)$	$G(\omega)$	
1	$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\frac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\frac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\operatorname{rect}\left(\frac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi}\operatorname{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Table 3.2
Fourier Transform Operations

Operation	$g(t)$	$G(\omega)$
Addition	$g_1(t)+g_2(t)$	$G_1(\omega)+G_2(\omega)$
Scalar multiplication	$kg(t)$	$kG(\omega)$
Symmetry	$G(t)$	$2\pi g(-\omega)$
Scaling	$g(at)$	$\frac{1}{\|a\|}G\left(\frac{\omega}{a}\right)$
Time shift	$g(t-t_0)$	$G(\omega)e^{-j\omega t_0}$
Frequency shift	$g(t)e^{j\omega_0 t}$	$G(\omega-\omega_0)$
Time convolution	$g_1(t)*g_2(t)$	$G_1(\omega)G_2(\omega)$
Frequency convolution	$g_1(t)g_2(t)$	$\frac{1}{2\pi}G_1(\omega)*G_2(\omega)$
Time differentiation	$\frac{d^n g}{dt^n}$	$(j\omega)^n G(\omega)$
Time integration	$\int_{-\infty}^{t} g(x)\,dx$	$\frac{G(\omega)}{j\omega}+\pi G(0)\delta(\omega)$